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Stochastic Schrödinger Equation from an Interaction with the Environment

Z. Haba[1]

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We consider a class of models describing a quantum oscillator in interaction with an environment. We show that models of continuous spontaneous localization based on a stochastic Schrödinger equation can be derived as an approximation to purely deterministic Hamiltonian systems.

1. INTRODUCTION

Conventional quantum mechanics assumes a sharp division of the world into the microworld of the system and the macroworld of the measuring devices. Such a theory can be considered as a theory of an ideal measurement. However, in contemporary mesoscopic physics such a division may be not adequate. We can imagine a direct observation of a mesoscopic body interacting with an invisible microparticle system. Such a record cannot be considered as a measurement in conventional quantum mechanics (e.g., the behavior of the mesoscopic body can be reversible). Nevertheless, it gives us some information about the microsystem. Such a situation inspires the study of the influence of a large (possibly infinite) system on the behavior of a single particle. It seems that together with an investigation of the classical limit of quantum mechanics done at the level of the wave function a study of the classical limit of the quantum theory of measurement is needed as well. It is well known that we encounter difficulties with an interpretation of quantum mechanics if it is applied to macroscopic bodies. However, it may be that the troublesome interference principle has no observational consequences when applied to realistic macroscopic bodies [apart from interesting situations on the boundary of the quantum and classical worlds (Leggett, 1980; Caldeira

[1] Institute of Theoretical Physics, University of Wroclaw, Wroclaw, Poland.

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and Leggett, 1983)]. Research in this field (which could in general be called the problem of decoherence) has increased recently (Zurek, 1982, 1991; Unruh and Zurek, 1989; Gell-Mann and Hartle, 1993; Dowker and Halliwell, 1992; Omnes, 1990; Joos and Zeh, 1985; Joos, 1987). The problem of decoherence seems to be closely related (Diosi *et al.*, 1995) to another old problem of quantum measurement theory, that of the reduction of the wave packet. There has recently appeared a conceptually simple theory of the wave packet reduction (Ghirardi *et al.*, 1990; Pearl, 1989; Gisin and Percival, 1992). In this theory the wave packet reduction is achieved through the addition of noise to the Schrödinger equation. Such a theory could be considered either as a phenomenological description of experiments (Joos and Zeh, 1985; Joos, 1987), or as a fundamental modification of quantum mechanics which is supposed to describe both microworld and macroworld (Ghirardi *et al.*, 1986, 1990; Pearl, 1989).

In this paper we do not intend to enter into basic epistemological problems. We investigate some simple models of an interaction of an oscillator with a reservoir [the simplest one is a textbook example (Louisell, 1973; Haken, 1986)]. The model is usually considered as a typical application of the quantum Langevin equation. We have shown (Haba, 1993, 1994a) that quantum dynamics can be expressed in terms of conventional diffusion processes (time-ordering replaces the noncommutativity). Such a stochastic description can be considered as a mathematical version of the Feynman path integral (Haba, 1994b). We think that such an approach is especially useful for a description of the dynamics of a subsystem. We can investigate the dynamics of a subsystem through an exact elimination of the coordinates of the environment [such a description is well known for classical stochastic systems (Zwanzig, 1973)]. In principle, this is the same strategy as suggested first by Feynman and Vernon (1963) and applied, e.g., by Leggett (1980), Caldeira and Leggett (1983), Zurek (1982), and Unruh and Zurek (1989). However, instead of using the path integral, we eliminate the coordinates of the environment on the level of stochastic equations. We get (without any approximation) closed equations for a particle moving in a reservoir. Our approach is based on the classical probability applied as a technical tool to standard quantum mechanics.

It comes out from our study that a noise which initially enters the calculations as a technical tool acquires the properties and functions of a physical noise. The Brownian motion, which initially is a realization of the sum over paths describing via the Feynman formula the unitary Schrödinger evolution, acquires an interpretation as a dissipation when we pass to the subsystem. There is still another source of noise (which we call a deterministic noise). The reservoir degrees of freedom evolving according to the deterministic Schrödinger dynamics in the limit of an infinite number of degrees of

freedom model quite well the white noise. This model of noise is similar to the mechanical model of Brownian motion discussed by Ford *et al.* (1965). We show that the Brownian noise together with the deterministic noise give in a proper limit the stochastic Schrödinger equation. As a consequence of the stochastic Schrödinger equation we obtain a model of a continuous spontaneous localization (Ghirardi *et al.*, 1990; Pearl, 1989). We show that in the energy representation the non-diagonal density matrix elements decay exponentially to zero. A localization in momentum (or space) is shown to hold true only in a limit of small (resp. large) oscillator frequency.

2. STOCHASTIC REPRESENTATION OF QUANTUM DYNAMICS

In Haba (1993, 1994a,b) we described the unitary Hamiltonian time evolution of pure states in terms of the Brownian motion. This formalism can be considered as a mathematical version of the Feynman path integral. On a formal level the method consists in the replacement of the integral over real paths $q(\cdot)$ by an integral over complex paths $\sqrt{i}q(\cdot)$ (where i is the imaginary unit). As a next step we change variables in the functional integral. The change of variables is expressed as a solution of a stochastic differential equation (depending on the Brownian motion). Then, the non-Gaussian functional integral can be reduced to the Gaussian integral over solutions of the stochastic equation. The aim of this formalism is to express in an explicit form the solution of the Schrödinger equation. Hence, it is completely equivalent to standard quantum mechanics.

Let U_t ($t \geq 0$) be a unitary Schrödinger evolution determined by the Hamiltonian

$$H = -\hbar^2 \frac{1}{2m} \Delta + V \tag{1}$$

Assume we know $|\chi\rangle_t = U_t|\chi\rangle$ [$\chi(x)$ will denote $|\chi\rangle$ in the coordinate representation, where $x \in R^n$]. Let the initial condition ψ for the Schrödinger equation be of the form $\psi(x) = \chi(x)\phi(x)$, where ϕ is an analytic function. Then the solution ψ_t of the Schrödinger equation with the initial condition ψ can be expressed as $\chi_t\phi_t$, where ϕ_t is the solution of the equation

$$\partial_t\phi_t = \frac{i\hbar}{2m} \Delta\phi_t + \frac{i\hbar}{m} \chi_t^{-1} \nabla\chi_t \nabla\phi_t \tag{2}$$

with the initial condition ϕ.

We express the solution of equation (2) by a stochastic process (Haba, 1993) [for time-dependent diffusions see Freidlin (1985)]. Then the unitary

Schrödinger evolution generated by the Hamiltonian (1) can be expressed in the form

$$(U_t\psi)(x) = \chi_t(x)E[\phi(q_t(x))] \tag{3}$$

where $q_t(x)$ is a complex diffusion process starting at $t = 0$ from x and solving the stochastic differential equation (here $0 \leq \tau \leq t$)

$$dq_\tau = \frac{i\hbar}{m}\chi_{t-\tau}^{-1}\nabla\chi_{t-\tau}(q_\tau)\,d\tau + \lambda\sigma\,db_\tau \tag{4}$$

where

$$\lambda = \frac{1}{\sqrt{2}}(1 + i)$$

and

$$\sigma = \left(\frac{\hbar}{m}\right)^{1/2}$$

The Brownian motion b_t is defined as the Gaussian process with independent increments and the variance

$$E[b_t^2] = t$$

In order to express the solution of the Schrödinger equation for negative time we can apply the complex conjugation

$$\overline{U_t\bar{\psi}} = \psi_{-t}$$

The action of arbitrary operators on states as well as correlation functions in arbitrary states can now be expressed as expectation values with respect to the Brownian motion (Haba, 1993). For example,

$$\begin{aligned}\langle\chi|F(x_t)G(x)|\chi\rangle &= \langle\chi|U_t^+F(x)U_tG(x)|\chi\rangle \\ &= E\left[\int dx\,|\chi_t(x)|^2F(x)G(q_t(x))\right]\end{aligned} \tag{5}$$

When $F = 1$, then equation (5) expresses a generalization of the notion of an invariant measure to time-inhomogeneous processes [if χ is a stationary state, then $|\chi(x)|^2$ is exactly the invariant measure for q_t (Freidlin, 1985)], i.e.,

$$E\left[\int dx\,|\chi_t(x)|^2G(q_t(x))\right] = \int dx\,|\chi(x)|^2G(x)$$

Let us consider as an example a model of independent oscillators with

$m = 1$ (here and later we do not specify the number of oscillators; it can be infinite):

$$H_0 = -\frac{\hbar^2}{2}\sum_k \frac{\partial^2}{\partial x_k^2} + \sum_k \frac{1}{2}\omega_k^2 x_k^2 \tag{6}$$

The ground-state solution of the Schrödinger equation with the Hamiltonian (6) reads

$$\chi_0(x) = \exp\left(-\frac{1}{2\hbar}\sum_k \omega_k x_k^2\right) \tag{7}$$

The stochastic equation (4) takes the form

$$dq_k = -i\omega_k q_k\, dt + \lambda\sigma\, db_k \tag{8}$$

The solution of equation (8) with the initial condition x reads

$$q_k(s) = \exp(-i\omega_k s)\, x_k + \lambda\sigma \int_0^s \exp(-i\omega_k(s-\tau))\, db_k(\tau) \tag{9}$$

3. AN INTERACTION WITH THE ENVIRONMENT

The formalism of Section 2 can easily be applied to Hamiltonians with explicitly known χ_t. An example of such a class of models are Hamiltonians (often encountered in quantum optics) of the form

$$H_S = \sum_{\mu\nu} h_{\mu\nu}(A, A^+)A_\mu^+ A_\nu \tag{10}$$

where A_μ is the annihilation operator, A_μ^+ the creation operator, and the Greek index indicates the space coordinate. In the case of the Hamiltonian (10) the harmonic oscillator's ground state (7) is also the ground state of H_S. Hence, we may take χ_0 as χ_t in equation (3).

We add to H_S the coupling to the reservoir. In this paper we restrict ourselves to the simplest one-dimensional model corresponding to $h = 1$. In such a case we obtain a standard model of an interaction with the environment [see Louisell (1973) and Haken (1986) and for a more recent review Ford *et al.* (1988)]

$$H = \hbar\omega_0 A^+A + \sum_k \hbar\omega_k a_k^+ a_k + \sum_k g_k A^+ a_k + \overline{g_k} a_k^+ A \tag{11}$$

Using the Schrödinger representation of the creation and annihilation operators

$$a = \left(\frac{\hbar}{2\omega}\right)^{1/2}\frac{\partial}{\partial x} + \left(\frac{\omega}{2\hbar}\right)^{1/2} x$$

we express H in the form (1). It is clear that the product of oscillator ground states is the ground state for H. In order to simplify the discussion we assume that

$$g_k = iv_k$$

are purely imaginary [if g_k had a real part, this would change only the form of the noise in the stochastic equation (4)]. Then, taking as χ_t in equation (4) the ground state (of Q and q_k oscillators), we obtain the following equations:

$$dq_k = -i\omega_k q_k\, dt + v_k\left(\frac{\omega_0}{\omega_k}\right)^{1/2} Q\, dt + \lambda\sigma\, db_k \tag{12}$$

$$dQ = -i\omega_0 Q\, dt - \sum_k v_k\left(\frac{\omega_k}{\omega_0}\right)^{1/2} q_k\, dt + \lambda\sigma\, db \tag{13}$$

We solve first equation (12) for q_k. We obtain

$$\begin{aligned} q_k(t) = {} & \exp(-i\omega_k t)\, x_k \\ & + v_k\left(\frac{\omega_0}{\omega_k}\right)^{1/2} \int_0^t \exp(-i\omega_k(t-s))Q(s)\, ds \\ & + \lambda\sigma \int_0^t \exp(-i\omega_k(t-s))\, db_k(s) \end{aligned} \tag{14}$$

Let us introduce the notation

$$N_{\mathrm{D}}(s) = \sum_k v_k x_k\left(\frac{\omega_k}{\omega_0}\right)^{1/2} \exp(-i\omega_k s) \tag{15}$$

and

$$N_{\mathrm{R}}(s) = \lambda\sigma \sum_k v_k\left(\frac{\omega_k}{\omega_0}\right)^{1/2} \int_0^s \exp(-i\omega_k(s-\tau))\, db_k(\tau) \tag{16}$$

corresponding to a "deterministic" and a random noise, respectively. We are interested in a computation of correlation functions in equation (5) with χ_t defined in equation (7). In such computations the x_k play the role of independent Gaussian random variables with the covariance

$$E[x_k x_r] = \frac{\hbar}{2\omega_k}\delta_{kr} \tag{17}$$

Hence, the noise N_{D} is a Gaussian complex stochastic process with the covariance

$$E[N_D(s)N_D(\tau)] = \frac{\hbar}{2\omega_0}\sum_k v_k^2 \exp(-i\omega_k(\tau + s)) \tag{18}$$

$$E[\overline{N_D(s)}N_D(\tau)] = \frac{\hbar}{2\omega_0}\sum_k v_k^2 \exp(-i\omega_k(\tau - s))$$

This noise is of the same order as the N_R noise (16). Next, let us denote

$$K(s) = \sum_k v_k^2 \exp(-i\omega_k s) \tag{19}$$

We insert the solution q_t of equation (12) into equation (13) for Q. We obtain an equation describing an oscillator moving in a reservoir

$$dQ = -i\omega_0 Q\, d\tau - \int_0^\tau K(\tau - s)Q(s)\, ds\, d\tau - N_D(\tau)\, d\tau \tag{20}$$
$$- N_R(\tau)\, d\tau + \lambda\sigma\, db$$

It can be seen from equation (19) that $K(s)$ is positive for small s, hence it fulfills the role of a dissipation. N_D, N_R, and b add to a random external force.

4. STOCHASTIC SCHRÖDINGER EQUATION FROM AN INTERACTION WITH THE ENVIRONMENT

In Leggett (1980), Caldeira and Leggett (1983), Zurek (1982), Unruh and Zurek (1989), Gell-Mann and Hartle (1993), Dowker and Halliwell (1992), and Omnes (1990) the classical behavior of quantum systems of the type (11) (in particular the decoherence) is explained by means of the Feynman–Vernon influence functional. This functional results from an average over environmental degrees of freedom. We could derive it explicitly from the solution of equation (20). There is another theory of decoherence. Ghirardi *et al.* (1990), Pearl (1989), and Gisin and Percival (1992) explain the emergence of classical properties through the addition of some stochastic terms to the Schrödinger equation. In spite of the ardent opposition of the two groups [see the letters in *Physics Today* (1993)], we would like to reconcile the two approaches. We restrict ourselves here to the model (11). In equation (20) the time evolution of the coordinate Q is influenced by the environment through the additional noise terms N_R and N_D. The term N_R [equation (16)] can be considered as a randomness resulting from the Feynman sum over paths (Haba, 1994b) (weighted sum over paths instead of the classical extremal path). The "deterministic" noise N_D [equation (15)] is produced by the environment of oscillators evolving in a deterministic way according to the Schrödinger evolution determined by the Hamiltonian (11). We can show that similarly as in the model of Ford *et al.* (1965), the

environment can influence the system as a random force of the form of the white noise. We note first that if in equation (18) $\nu_k \approx$ const and $\omega_k \sim k$, then the noise $N_D(s)$ is strongly correlated only at coinciding arguments. Under these assumptions on ν_k and ω_k the dissipation kernel $K(s)$ in equation (19) can now be approximated by (such a behavior of K is sometimes called the Ohm dissipation)

$$K(s) = a\delta(s)$$

where a is a positive constant. In fact, we get such a formula if $\omega_k = k\omega$, where $k \in [0, \infty]$, and $\nu_k^2 = (a/\pi)\omega\Delta k$. In such a case the sum over infinitesimal Δk can be replaced by an integral

$$\int_0^\infty dk \exp(-ik\omega s) = \frac{\pi}{\omega}\delta(s) - \frac{i}{s\omega}$$

The term s^{-1} is considered small in comparison to $\delta(s)$. The corresponding choice of ν_k^2 leads to the following approximation to N_D [again the term $(t-s)^{-1}$ is neglected]

$$E[N_D(s)N_D(t)] = 0 \tag{21}$$
$$E[\overline{N_D(s)}N_D(t)] = \hbar\epsilon\delta(t-s)$$

where

$$\epsilon = \frac{a}{2\omega_0}$$

So, N_D is the white noise. We denote the corresponding Brownian motion by B_D, i.e.,

$$\frac{dB_D}{ds} = N_D$$

Next, let us consider the "quantum" noise N_R [equation (16)] in more detail. Let us compute its correlation functions:

$$E[N_R(s)N_R(\tau)] = \frac{\hbar}{2\omega_0}\sum_k \nu_k^2(\exp(-i\omega_k|\tau - s|) - \exp(-i\omega_k(\tau + s)))$$

$$E[\overline{N_R(s)}N_R(\tau)] = \hbar \min(s, \tau)\frac{1}{\omega_0}\sum_k \omega_k\nu_k^2 \exp(i\omega_k(s-\tau)) \tag{22}$$

In the approximation of the Ohm dissipation we get by the same arguments as applied in the derivation of equation (21)

$$E[N_R(s)N_R(\tau)] = \hbar\epsilon\delta(s-\tau) \tag{23}$$

Equation (23) means that N_R is the real white noise [the second of the expectation values in (22) does not enter any subsequent formulas]. We denote the integral of N_R by B_R (the real Brownian motion independent of B_D). Equation (20) now reads

$$dQ = -i\omega_0 Q\, ds - aQ\, ds - dB_D(s) - dB_R(s) + \lambda\sigma\, db(s) \tag{24}$$

[all the noise terms on the right-hand side of equation (24) are independent].

The dynamics of a subsystem is usually investigated in terms of the time evolution of its density matrix. We define the density matrix of a subsystem (with coordinates X) as

$$\rho_P(X, X') = \int \prod_k dx_k\, \rho(X, x; x, X') \tag{25}$$

We make the usual assumption that initially the system and the environment are separated, i.e., ρ is of the product form

$$\rho(X, x; x, X') = \exp\left(-\frac{\omega_0}{2\hbar}(X^2 + X'^2)\right) \overline{\nu(X)}\nu(X')\rho_0(x, x)$$

We have expressed the initial wave function of the subsystem as a product of the ground-state wave function [with $m = 1$] and arbitrary $\nu(X)$ [this is a proper choice for the class of models (10)–(11)]. We choose for ρ_0 the probability density in the ground state (7). This is a reasonable assumption for the reservoir at zero temperature [the stochastic formalism of Section 2 at positive temperature is discussed in Haba (1995)].

We can express in a simple way the evolution of subsystem's density matrix in terms of the Brownian motion

$$\begin{aligned}\rho_P(t; X, X') &\\ &= \int \prod_k dx_k \exp\left(-\frac{\omega_0}{2\hbar}(X^2 + X'^2)\right) |\chi_0(x)|^2 \\ &\times \overline{E[\nu(Q_t(X))]}E[\nu(Q_t(X'))]\end{aligned} \tag{26}$$

For the model (11) the stochastic process Q is the solution of equation (20). When we apply the definition of the noise N_D [equations (15) and (17)] then the integration over x_k can be expressed as an average over N_D,

$$\begin{aligned}\rho_P(t; X, X') \simeq \exp\left(-\frac{\omega_0}{2\hbar}(X^2 + X'^2)\right) \\ \times E_D[\overline{E[\nu(Q_t(X))]}E[\nu(Q_t(X'))]]\end{aligned} \tag{27}$$

where $E[..]$ means the expectation value with respect to the Brownian motion b and E_D means that the integration over x is replaced by an expectation value with respect to the complex white noise N_D. An application of the standard Ito stochastic calculus (see, e.g., Gikhman and Skorohod, 1972) shows that $v(Q_t)$ satisfies the Ito equation

$$dv = \frac{\partial v}{\partial Q}(-i\omega_0 Q\, ds - aQ\, ds - dB_D(s) - dB_R + \lambda\sigma\, db) + \epsilon\frac{\hbar}{2}\frac{\partial^2 v}{\partial Q^2}(1+i)\, ds \tag{28}$$

The wave function $\psi(X) = \exp(-\omega_0 X^2/2\hbar)v(X)$ satisfies the stochastic Schrödinger equation

$$\exp\left(-\omega_0\frac{Q^2}{2\hbar}\right) d\exp\left(\omega_0\frac{Q^2}{2\hbar}\right)\psi = -\frac{i}{\hbar}H_0\psi\, ds - \frac{\epsilon}{\hbar}H_0\psi\, ds - L\psi(dB_D(s) + dB_R(s)) + \lambda\sigma L\psi\, db(s) \tag{29}$$

where H_0 is the Hamiltonian of the oscillator [the formula (6) for the ω_0-oscillator]

$$L = \frac{\partial}{\partial X} + \frac{\omega_0}{\hbar}X$$

We can derive the master equation for ρ_P from equation (27) without any further approximation. For this purpose we differentiate ρ_P in equation (27) and apply equation (29). Then, elementary rules of the stochastic calculus (Gikhman and Skorohod, 1972) lead to the formula

$$\partial_t\rho_P = -\frac{i}{\hbar}[H_0, \rho_P] - \frac{\epsilon}{\hbar}(H_0\rho_P + \rho_P H_0) + \hbar\epsilon L\rho_P L^+ \tag{30}$$

We have obtained the master equation exactly in the Lindblad form (Lindblad, 1975; Gorini *et al.*, 1976) as

$$H_0 = \frac{\hbar^2}{2}L^+L$$

The approximation which leads to the stochastic equation (29) is a Markovian approximation implying linear differential equations for the density matrix. The Lindblad form of the evolution equation ensures that the evolution preserves the normalization of the density matrix and its positivity.

If $\rho_P(t)$ is of the form (27) then at fixed $\hbar$ and $t \to \infty$ $\rho_P(t) \to |\chi_0\rangle\langle\chi_0|$ where $|\chi_0\rangle$ is the oscillator ground state. Let us consider another limit when

the quantum numbers tend to infinity while $\hbar$ tends to zero in such a way that energies remain finite. We consider equation (30) in the energy representation. Let $|k\rangle$ be the kth eigenstate of H_0. Then equation (30) for matrix elements reads (for a derivation it is sufficient to notice that L is proportional to the annihilation operator)

$$\partial_t\langle j|\rho_P|k\rangle = -i\omega_0(j-k)\langle j|\rho_P|k\rangle - \epsilon\omega_0(j+k)\langle j|\rho_P|k\rangle + 2\epsilon\omega_0\sqrt{(j+1)(k+1)}\langle j+1|\rho_P|k+1\rangle \tag{31}$$

For large j and k for operators whose matrix elements have a limit $\hbar \to 0$ $\langle j|\rho_P|k\rangle \simeq \langle j+1|\rho_P|k+1\rangle$. Applying this approximation to the last term in equation (31), we obtain the solution

$$\langle j|\rho_P(t)|k\rangle \simeq \exp\left[-\frac{i}{\hbar}(E_j - E_k)t - \frac{\epsilon}{\hbar}(\sqrt{E_j} - \sqrt{E_k})^2 t\right]\langle j|\rho_P(0)|k\rangle \tag{32}$$

where we denoted by E_k the kth energy eigenvalue. Equation (32) shows that the interference disappears on a time scale proportional to the inverse of the energy difference. Hence, there is no interference of states with macroscopically distinguishable energies. There is still another interpretation of the result. Assume that in the framework of quantum mechanics of closed systems with the conventional wave packet reduction postulate an apparatus performed a measurement of the energy eigenvalues, but the results are unknown. Then the state of the system after the measurement is described by the solution (32) at large time. In this sense the interaction with an environment produces the same effect as a measurement.

If we consider equation (30) in the momentum representation and assume $\omega_0 X^2 \simeq 0$, so that we can neglect the oscillator potential energy in comparison with the kinetic energy, then from equation (30) we get approximately

$$\partial_t\langle p|\rho_P|p'\rangle \simeq -\frac{\epsilon}{2\hbar}(p-p')^2\langle p|\rho_P|p'\rangle \tag{33}$$

Equation (33) means a localization in momentum. If on the other hand we assume that $\omega_0 X^2$ is large in comparison with the kinetic energy, then neglecting the momenta in equation (30), we get in the position representation

$$\partial_t\langle X|\rho_P|X'\rangle \simeq -\frac{\epsilon\omega_0^2}{\hbar}(X-X')^2\langle X|\rho_P|X'\rangle \tag{34}$$

i.e., a decoherence in space.

5. SUMMARY AND DISCUSSION

We have discussed a description of the dynamics of a subsystem which clearly displays the dissipation and noise in the subsystem. It is shown in a

simple model that some classical properties can emerge in quantum systems from an interaction with a large (quantum) environment. In particular, we have derived a stochastic Schrödinger equation discussed in the theory of measurement.

The theory of an interaction of microscopic and macroscopic bodies based on the stochastic Schrödinger equation has some appealing features. It describes the dynamics of decoherence and localization in a model-independent way. However, we think that the microworld and macroworld will meet in conventional quantum mechanics. In such a case the problem of the consistency of the deterministic (in the sense of the Schrödinger time evolution) and stochastic evolutions will emerge. This was our motivation for an investigation of the stochastic Schrödinger equation as a consequence of a deterministic model. The stochastic Schrödinger equation derived in Section 4 seems to be a realistic approximation to the time evolution in a reservoir. Surprisingly, it resembles a model of Ghirardi *et al.* (1990) and Pearl (1989) for a continuous spontaneous localization during the energy measurement. A measurement of another observable needs another environment which in general would be difficult to define explicitly. However, it is important to enquire whether we can reduce the number of basic principles of quantum mechanics without any modification of the Schrödinger equation and the superposition principle.

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